

# Dynamics of Bianchi $I$ Universe with Magnetized Anisotropic Dark Energy

M. Sharif <sup>\*</sup> and M. Zubair <sup>†</sup>

Department of Mathematics, University of the Punjab,  
Quaid-e-Azam Campus, Lahore-54590, Pakistan.

## Abstract

We study Bianchi type  $I$  cosmological model in the presence of magnetized anisotropic dark energy. The energy-momentum tensor consists of anisotropic fluid with anisotropic EoS  $p = \omega\rho$  and a uniform magnetic field of energy density  $\rho_B$ . We obtain exact solutions to the field equations using the condition that expansion is proportional to the shear scalar. The physical behavior of the model is discussed with and without magnetic field. We conclude that universe model as well as anisotropic fluid do not approach isotropy through the evolution of the universe.

**Keywords:** Electromagnetic Field; Dark Energy; Anisotropy.

**PACS:** 04.20.Jb; 04.20.Dw; 04.40.Nr; 98.80.Jk

## 1 Introduction

Recent cosmological observations contradict the matter dominated universe with decelerating expansion indicating that our universe experiences accelerated expansion. The accelerating expansion of the universe is driven by mysterious energy with negative pressure known as *Dark Energy* (DE). The evidence of the existence of DE comes from the Supernova observations [1, 2] and

---

<sup>\*</sup>msharif@math.pu.edu.pk

<sup>†</sup>mzubairkk@gmail.com

other observations such as cosmic microwave background (CMB) anisotropies measured with WMAP satellite [3] and large scale structure [4]. These observations suggest that nearly two-third of our universe consists of DE and the remaining consists of relativistic dark matter and baryons [5].

In spite of all the observational evidences, the nature of DE is still a challenging problem in theoretical physics. A variety of possible solutions such as cosmological constant [6], quintessence [7], phantom field [8], tachyon field [9], quintom [10], and the interacting DE models like Chaplygin gas [11], holographic models [12] and braneworld models [13] etc. have been proposed to interpret accelerating universe. However, none of these models can be regarded as being entirely convincing so far.

Recently, many authors have studied the Bianchi type  $I$  model in the presence of anisotropic DE. Rodrigues [14] constructed a Bianchi type  $I$   $\Lambda$ CDM cosmological model whose DE component preserves non-dynamical character but yields anisotropic vacuum pressure. Koivisto and Mota [15, 16] proposed a different approach to resolve the CMB anisotropy problem; the earlier isotropy of the universe could be distorted by the direction dependent acceleration of the later universe. Koivisto and Mota [16] investigated the Bianchi  $I$  cosmological model containing interacting DE fluid with non-dynamical anisotropic EoS and perfect fluid component. They suggested that if the EoS is anisotropic, the expansion rate of the universe becomes direction dependent at late times and cosmological models with anisotropic EoS can explain some of the observed anomalies in CMB.

Mota et al. [17] explored the possibility of using the cosmological observation to probe and constrain an imperfect DE fluid. They concluded that a perfect fluid representation of DE might ultimately turn out to be a phenomenologically sufficient description of all the observational consequences of DE. However, one cannot exclude the possibility of imperfectness in DE. Akarsu and Kilinc [18, 19] suggested that anisotropic fluid must not necessarily promote anisotropy in the expansion whereas such fluid may also act to support isotropic behavior of the universe. It has been shown [18] that anisotropic Bianchi  $I$  model in the presence of perfect fluid and minimally interacting DE shows isotropic behavior for the earlier times of the universe.

Primordial magnetic fields can have a significant impact on the CMB anisotropy depending on the direction of field lines [20, 21]. Many people have investigated the influence of magnetic field on the dynamics of universe by analyzing anisotropic Bianchi models. Milaneschi and Fabbri [22] studied the anisotropy and polarization properties of CMB radiation in ho-

mogeneous Bianchi  $I$  cosmological model. Jacobs [23] explored the effects of a uniform, primordial magnetic field on Bianchi type  $I$  cosmological model. He concluded that the primordial magnetic field produced large expansion anisotropies during the radiation-dominated phase but it had negligible effect during the dust-dominated phase. King and Coles [21] discussed the dynamics of magnetized axisymmetric Bianchi  $I$  universe with vacuum energy. He examined the behavior of scale factors perpendicular and parallel to the field lines. Roy et al. [24] investigated Bianchi type  $I$  cosmological models containing perfect fluid and magnetic field directed along  $x$  axis. Exact solutions are obtained using the condition that expansion is proportional to shear scalar.

In this paper, we would like to investigate the effects of magnetic field on the dynamics of anisotropic Bianchi  $I$  model in the presence of anisotropic DE. The paper has the following format. In section 2, we present anisotropic Bianchi type  $I$  model and formulate the dynamical field equations which describe the evolution of the universe. In section 3, we obtain exact solution to the field equations and discuss the physical properties of the solution. Section 4 contains a brief discussion related to two special cases for  $\beta = 0$  and  $m = 1$ . Finally, in section 5, we summarize the results.

## 2 Bianchi $I$ Model and the Field Equations

The line element for the spatially homogeneous, anisotropic and LRS Bianchi type  $I$  spacetime is given by

$$ds^2 = dt^2 - A^2(t)dx^2 - B^2(t)(dy^2 + dz^2), \quad (1)$$

where the scale factors  $A$  and  $B$  are functions of cosmic time  $t$  only. For  $A(t) = B(t) = a(t)$ , this reduces to the flat FRW spacetime. This spacetime has one transverse direction  $x$  and two equivalent longitudinal directions  $y$  and  $z$ . We assume that the universe is filled with anisotropic fluid, and that there is no electric field while the magnetic field is oriented along  $z$  axis. The scale factor  $A(t)$  is in the transverse direction, perpendicular to magnetic field while  $B(t)$  is along the direction of field lines. King and Coles [21] and Jacobs [23] used the magnetized perfect fluid energy-momentum tensor to discuss the effects of magnetic field on the evolution of the universe.

Here we take a more general energy-momentum tensor for the magnetized

anisotropic DE fluid in the following form

$$T_\mu^\nu = \text{diag}[\rho + \rho_B, -p_x + \rho_B, -p_y - \rho_B, -p_z - \rho_B], \quad (2)$$

where  $\rho$  is the energy density of the fluid;  $p_x$ ,  $p_y$  and  $p_z$  are pressures on  $x$ ,  $y$  and  $z$  axes respectively and  $\rho_B$  stands for energy density of magnetic field. The anisotropic fluid is characterized by the EoS  $p = \omega\rho$ , where  $\omega$  is not necessarily constant [25]. From Eq.(2), we have

$$T_\mu^\nu = \text{diag}[\rho + \rho_B, -(\omega + \delta)\rho + \rho_B, -(\omega + \gamma)\rho - \rho_B, -(\omega + \gamma)\rho - \rho_B], \quad (3)$$

where  $\omega_x = \omega + \delta$ ,  $\omega_y = \omega + \gamma$ , and  $\omega_z = \omega + \gamma$  are the directional EoS parameters on  $x$ ,  $y$  and  $z$  axes respectively.  $\delta$  and  $\gamma$  are the deviations from  $\omega$  on  $x$  and  $y$ ,  $z$  axes respectively. If the deviation parameters are zero, then Eq.(2) represents the energy-momentum tensor for the isotropic fluid and magnetic field [21]. For zero magnetic field, Eq.(2) is reduced to the energy-momentum tensor of anisotropic fluid [18].

The Einstein field equations are given by

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = T_{\mu\nu}, \quad (4)$$

where  $R_{\mu\nu}$  is the Ricci tensor,  $R$  is the Ricci scalar and  $T_{\mu\nu}$  is the energy-momentum tensor for magnetized anisotropic fluid. For the Bianchi type  $I$  spacetime, the field equation take the form

$$2\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}^2}{B^2} = \rho + \rho_B, \quad (5)$$

$$2\frac{\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} = -(\omega + \delta)\rho + \rho_B, \quad (6)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} = -(\omega + \gamma)\rho - \rho_B, \quad (7)$$

where dot denotes derivative with respect to time. The energy conservation equation,  $T_{\nu;\mu}^\mu = 0$ , leads to two equations for the anisotropic fluid and magnetic field [21]

$$\dot{\rho} + (1 + \omega)\rho\left(\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B}\right) + \rho\left(\delta\frac{\dot{A}}{A} + 2\gamma\frac{\dot{B}}{B}\right) = 0, \quad (8)$$

$$\rho_B = \frac{\beta}{B^4}. \quad (9)$$

The conservation equation for the anisotropic fluid can be decomposed into two parts,

$$T_{\nu;\mu}^{\mu} = \dot{T}_{\nu;\mu}^{\mu} + \tau_{\nu;\mu}^{\mu} = 0, \quad (10)$$

where  $\tau_{\nu;\mu}^{\mu}$  is the last term in Eq.(8) which arises due to the anisotropy in the fluid and  $\dot{T}_{\nu;\mu}^{\mu}$  represents the deviation free part of the  $T_{\nu;\mu}^{\mu}$ . Let us take [18]

$$\tau_{\nu;\mu}^{\mu} = \rho(\delta \frac{\dot{A}}{A} + 2\gamma \frac{\dot{B}}{B}) = 0. \quad (11)$$

Using this assumption in Eq.(8), we obtain conservation of the perfect fluid

$$\dot{T}_{\nu;\mu}^{\mu} = \dot{\rho} + (1 + \omega)\rho(\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B}) = 0. \quad (12)$$

Equation (11) is satisfied either  $\delta(t)$  and  $\gamma(t)$  are trivially zero or the ratio of expansion rate on  $x$  axis to the  $y$  axis is equal to  $-2\gamma/\delta$ . In order to obtain more general solution, the deviation parameter on the  $x$  axis  $\delta(t)$  is assumed to be [18]

$$\delta(t) = n \frac{2}{3} \frac{\dot{B}}{B} (\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B}) \frac{1}{\rho} \quad (13)$$

and hence the deviation parameter on  $y$  and  $z$  axes is given by

$$\gamma(t) = -n \frac{1}{3} \frac{\dot{A}}{A} (\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B}) \frac{1}{\rho}, \quad (14)$$

where  $\delta(t)$  and  $\gamma(t)$  are dimensionless parameters and  $n$  is the real dimensionless constant that parameterizes the deviation from EoS parameter. The anisotropy of the DE is measured using the relation  $(\delta(t) - \gamma(t))/\omega(t)$  and for  $n = 0$ , DE is found to be isotropic.

### 3 General Parameters and Solution of the Field Equations

Here we define some parameters for the Bianchi  $I$  model which are important in cosmological observations. The average scale factor and the volume are defined as

$$a = (AB^2)^{\frac{1}{3}}, \quad V = a^3 = AB^2. \quad (15)$$

The anisotropy parameter of the expansion is characterized by the mean and directional Hubble parameters and is defined as

$$\Delta = \frac{1}{3} \sum_{i=1}^3 \left( \frac{H_i - H}{H} \right)^2, \quad (16)$$

where

$$H = \frac{1}{3} (\ln V)^\cdot = \ln \dot{a} = \frac{1}{3} \left( \frac{\dot{A}}{A} + 2 \frac{\dot{B}}{B} \right),$$

is the mean Hubble parameter and  $H_i (i = 1, 2, 3)$  represent the directional Hubble parameters in the directions of  $x$ ,  $y$  and  $z$  axes respectively, and are given by

$$H_x = \frac{\dot{A}}{A}, \quad H_y = H_z = \frac{\dot{B}}{B}.$$

The anisotropy of the expansion results in isotropic expansion of the universe for  $\Delta = 0$ . The physical parameters like scalar expansion  $\Theta$ , shear scalar  $\sigma^2$  are given by

$$\Theta = u_{;a}^a = \frac{\dot{A}}{A} + 2 \frac{\dot{B}}{B}, \quad (17)$$

$$\sigma^2 = \frac{1}{2} \sigma_{ab} \sigma^{ab} = \frac{1}{3} \left[ \frac{\dot{A}}{A} - \frac{\dot{B}}{B} \right]^2. \quad (18)$$

It is mentioned here that any universe model becomes isotropic for the diagonal energy-momentum tensor when  $t \rightarrow +\infty$ ,  $\Delta \rightarrow 0$ ,  $V \rightarrow +\infty$  and  $T^{00} > 0$  ( $\rho > 0$ ) [19, 26].

In order to solve the field equations, we use a physical condition that expansion scalar is proportional to shear scalar. According to Throne [27], observations of velocity red shift relation for extragalactic sources suggest that Hubble expansion of the universe is isotropic within about 30% range approximately [28, 29] and red shift studies place the limit  $\frac{\sigma}{H} \leq .30$ , where  $\sigma$  is the shear and  $H$  is the Hubble constant. Collins [30] discussed the physical significance of this condition for perfect fluid and barotropic EoS in a more general case. In many papers [24, 31]-[33], this condition is proposed to find the exact solutions of cosmological models. It is given by

$$A = B^m, \quad (19)$$

where  $m \neq 1$  is a positive constant. Subtracting Eqs.(6) and (7), it follows that

$$\frac{\ddot{A}}{A} + \frac{\dot{A}\dot{B}}{AB} - \frac{\ddot{B}}{B} - \frac{\dot{B}^2}{B^2} = \frac{n}{3} \left( \frac{\dot{A}^2}{A^2} + 4 \frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}^2}{B^2} \right) - \frac{2\beta}{B^4}. \quad (20)$$

Applying the condition given in Eq.(19), we have

$$2\ddot{B} + \frac{2(3m^2 - 3 - nl)}{3(m-1)} \frac{\dot{B}^2}{B} = -\frac{4\beta}{B^3(m-1)}, \quad (21)$$

where  $l = m^2 + 4m + 4$  is a positive constant. Replacing  $\dot{B} = f(B)$ , it follows that

$$\frac{df^2}{dB} + \frac{2(3m^2 - 3 - nl)}{3(m-1)} \frac{f^2}{B} = -\frac{4\beta}{B^3(m-1)} \quad (22)$$

which has the solution

$$f^2 = \left( \frac{dB}{dt} \right)^2 = cB^{-\frac{2(3m^2 - 3 - nl)}{3(m-1)}} - \frac{6\beta B^{-2}}{(3m^2 - 3m - nl)}, \quad (23)$$

where  $c$  is a constant of integration. Thus the spacetime reduces to the form

$$\begin{aligned} ds^2 &= \left( \frac{dt}{dB} \right)^2 dB^2 - B^{2m}(t) dx^2 - B^2(t) (dy^2 + dz^2), \\ &= \frac{dT^2}{cT^{-\frac{2(3m^2 - 3 - nl)}{3(m-1)}} - \frac{6\beta T^{-2}}{(3m^2 - 3m - nl)}} - T^{2m} dx^2 - T^2 \\ &\quad \times (dy^2 + dz^2). \end{aligned} \quad (24)$$

where  $B = T$ ,  $x = X$ ,  $y = Y$ ,  $z = Z$ .

### 3.1 Some Physical Features of the Model

Now we discuss some physical aspects of the model (24). The directional and the mean Hubble parameters will become

$$\begin{aligned} H_x &= mH_y = m \left\{ cT^{-\frac{2(3m^2 + 3m - 6 - nl)}{3(m-1)}} - \frac{6\beta T^{-4}}{3m^2 - 3m - nl} \right\}^{1/2}, \\ H &= \frac{m+2}{3} \left\{ cT^{-\frac{2(3m^2 + 3m - 6 - nl)}{3(m-1)}} - \frac{6\beta T^{-4}}{3m^2 - 3m - nl} \right\}^{1/2}. \end{aligned} \quad (25)$$

We see that these quantities are found to be dynamical and take infinitely large values at  $T = 0$  for  $m \geq 2$ . The values of Hubble parameters  $H$ ,  $H_x$ ,  $H_y$

decrease with the increase in time and approach to zero as  $T \rightarrow \infty$  for  $n < \frac{3m^2+3m-6}{l}$ . The scale factors are found to be zero at  $T = 0$  and hence the model exhibits *point type* singularity. The volume of the universe model is given by

$$V = T^{m+2} \quad (26)$$

and anisotropy parameter of the expansion is found to be

$$\Delta = 2 \frac{(m-1)^2}{(m+2)^2}. \quad (27)$$

The expansion and shear scalar take the form

$$\Theta = 3H = (m+2) \left\{ cT^{\frac{-2(3m^2+3m-6-nl)}{3(m-1)}} - \frac{6\beta T^{-4}}{3m^2-3m-nl} \right\}^{1/2}, \quad (28)$$

$$\sigma = \frac{(m-1)^2}{3} \left\{ cT^{\frac{-2(3m^2+3m-6-nl)}{3(m-1)}} - \frac{6\beta T^{-4}}{3m^2-3m-nl} \right\}. \quad (29)$$

We note that spatial volume is zero at initial epoch and increases as  $T \rightarrow \infty$ . The expansion and shear scalar are infinite at  $T = 0$  and decreases with the increase in cosmic time. Thus the universe starts evolving with the zero volume at the initial epoch with infinite rate of expansion which slows down for the later times of the universe. The component of magnetic field reduces the expansion, shear scalar and Hubble parameters. The anisotropy parameter of the expansion is found to be constant and becomes zero at  $m = 1$  (it will be discussed as a special case). Thus the model does not approach to isotropy for the future evolution of the universe. The most general form of energy density is found by using Eqs.(5) and (16) as

$$\rho = 3H^2 \left(1 - \frac{\Delta}{2}\right) - \frac{\beta}{B^4}. \quad (30)$$

The energy density of the DE for the model (24) turns out to be

$$\rho = c(2m+1)T^{\frac{-2(3m^2+3m-6-nl)}{3(m-1)}} - \frac{\beta T^{-4}(3m^2+9m+6-nl)}{3m^2-3m-nl}. \quad (31)$$

The energy condition  $\rho \geq 0$  leads to

$$T^{\frac{-2(3m^2+3m-6-nl)}{3(m-1)}} \geq \frac{\beta T^{-4}(3m^2+9m+6-nl)}{c(2m+1)(3m^2-3m-nl)}.$$



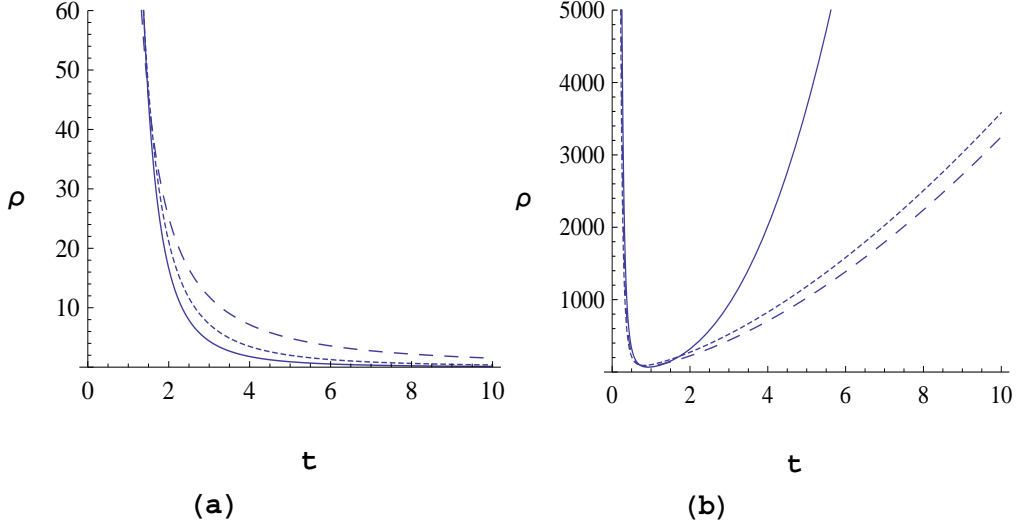


Figure 1: Plots of  $\rho$  verses cosmic time  $t$  (a)  $n < \frac{3m^2+3m-6}{l}$  (b)  $n > \frac{3m^2+3m-6}{l}$ , with varying values of  $m$  as follows: solid,  $m = 2$ ; dashed,  $m = 3$ ; dotted,  $m = 4$ .

This shows that energy density of the anisotropic DE is reduced by the magnetic field. This turns out to be infinite at the initial epoch, its value decreases with the increase in time and converges to zero as  $T \rightarrow \infty$  with the condition that  $n < \frac{3m^2+3m-6}{l}$  shown in Figure 1(a). However, for  $n > \frac{3m^2+3m-6}{l}$ , energy density decreases after big bang but it starts increasing and becomes infinite as  $T \rightarrow \infty$  shown in Figure 1(b).

Using Eqs.(25) and (31) in Eqs.(13) and (14), the deviation parameters  $\delta(T)$  and  $\gamma(T)$  become

$$\delta(T) = \frac{2n(m+2)(cT^{\frac{-2(3m^2+3m-6-nl)}{3(m-1)}} - \frac{6\beta T^{-4}}{3m^2-3m-nl})}{3(c(2m+1)T^{\frac{-2(3m^2+3m-6-nl)}{3(m-1)}} - \frac{\beta T^{-4}(3m^2+9m+6-nl)}{3m^2-3m-nl})}, \quad (32)$$

$$\gamma(T) = -\frac{nm(m+2)(cT^{\frac{-2(3m^2+3m-6-nl)}{3(m-1)}} - \frac{6\beta T^{-4}}{3m^2-3m-nl})}{3(c(2m+1)T^{\frac{-2(3m^2+3m-6-nl)}{3(m-1)}} - \frac{\beta T^{-4}(3m^2+9m+6-nl)}{3m^2-3m-nl})}. \quad (33)$$

The deviation free EoS parameter can be obtained by using the expressions

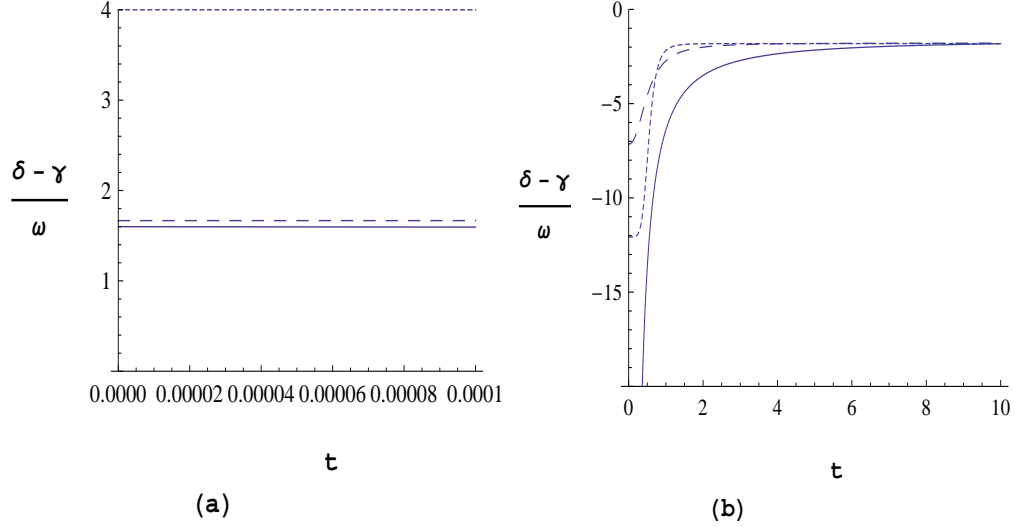


Figure 2: Plots of  $(\delta - \gamma)/\omega$  verses cosmic time  $t$  with  $n < \frac{3m^2+3m-6}{l}$  (a) for initial epoch (b) for future evolution, with varying values of  $m$  as follows: solid,  $m = 2$ ; dashed,  $m = 3$ ; dotted,  $m = 4$ .

for directional Hubble parameters and energy density in Eq.(12)

$$\omega(T) = -1 - \frac{\left( \frac{4\beta T^{-4}(3m^2+9m+6-nl)}{3m^2-3m-nl} - \frac{2c(1+2m)(3m^2+3m-6-nl)T^{\frac{-2(3m^2+3m-6-nl)}{3(m-1)}}}{3(m-1)} \right)}{(2+m)(c(1+2m)T^{\frac{-2(3m^2+3m-6-nl)}{3(m-1)}} - \frac{\beta T^{-4}(3m^2+9m+6-nl)}{3m^2-3m-nl})} \quad (34)$$

The anisotropy measure of anisotropic fluid is given by

$$\frac{(\delta - \gamma)}{\omega} = \frac{n(m+2)^3 \left( cT^{\frac{-2(3m^2+3m-6-nl)}{3(m-1)}} - \frac{6\beta T^{-4}}{(3m^2-3m-nl)} \right)}{\frac{c(1+2m)(3m^2+3m-6-2nl)}{(m-1)} T^{\frac{-2(3m^2+3m-6-nl)}{3(m-1)}} + \frac{3\beta(m-2)(3m^2+9m+6-nl)T^{-4}}{3m^2-3m-nl}} \quad (35)$$

The deviation parameters  $\delta(T)$  and  $\gamma(T)$  are found to be finite at  $t = 0$  and converges to  $4n(m+2)/(3m^2+9m+6-nl)$  and  $-2nm(m+2)/(3m^2+9m+6-nl)$  respectively as  $T \rightarrow \infty$  and  $n < \frac{3m^2+3m-6}{l}$ . The anisotropy measure of the DE  $(\delta - \gamma)/\omega$  is constant at  $t = 0$  shown in Figure 2(a) and converges to  $2n(2+m)^3/((2-m)(3m^2+9m+6-nl))$  for the later times

of the universe (Figure 2(b)) with the condition that  $n < \frac{3m^2-3m}{l}$  and vice versa for  $n > \frac{3m^2-3m}{l}$ . We note that the anisotropy of DE does not vanish throughout the evolution of the universe. Now we check the behavior of  $\omega$  for  $n < \frac{3m^2-3m}{l}$ . For the earlier times of the universe, the deviation free EoS parameter of the DE is given by  $\omega = -1 + \frac{2(3m^2+3m-6-nl)}{3(m^2+m-2)}$  which shows that expansion in the universe may be in the quintessence region after big bang. Also,  $\omega \rightarrow -1 + \frac{4}{m+2}$  as  $T \rightarrow \infty$ . Hence for later times,  $\omega$  represent EoS of cosmological constant for  $m = 2$  and  $\omega$  may result in quintessence region for  $m > 2$ . Thus the model represents the accelerating expanding universe.

## 4 Some Special Cases

Here we discuss the following two special cases, i.e., model with  $\beta = 0$  and model with  $m = 1$ .

### 4.1 Model with $\beta = 0$

In the absence of magnetic field i.e.,  $\beta \rightarrow 0$ , the model (24) reduces to the following form

$$ds^2 = \frac{dT^2}{cT^{-\frac{2(3m^2-3-nl)}{3(m-1)}}} - T^{2m}dx^2 - T^2(dy^2 + dz^2). \quad (36)$$

For this model, the directional, mean Hubble parameters and energy density of the DE reduce to the following form

$$\begin{aligned} H_x &= mH_y = c^{1/2}mT^{\frac{-(3m^2+3m-6-nl)}{3(m-1)}}, \\ H &= \frac{c^{1/2}(m+2)}{3}T^{\frac{-(3m^2+3m-6-nl)}{3(m-1)}}, \end{aligned} \quad (37)$$

$$\rho = c(2m+1)T^{\frac{-2(3m^2+3m-6-nl)}{3(m-1)}}. \quad (38)$$

We see that dynamical  $H$ ,  $H_x$ ,  $H_y$  and  $\rho$  are infinite for earlier times and converge to zero as  $T \rightarrow \infty$  provided that  $n < \frac{3m^2+3m-6}{l}$  and vice versa. The expansion and scalar are found to be

$$\Theta = 3H = \frac{c^{1/2}(m+2)}{3}T^{\frac{-(3m^2+3m-6-nl)}{3(m-1)}}, \quad (39)$$

$$\sigma = \frac{c(m-1)^2}{3}T^{\frac{-2(3m^2+3m-6-nl)}{3(m-1)}}. \quad (40)$$

The expansion in the universe is infinite at the initial epoch and decreases with the increase in time for  $n < \frac{3m^2+3m-6}{l}$ . However, if  $n > \frac{3m^2+3m-6}{l}$  then the universe starts expanding at  $T = 0$  and expands indefinitely as  $T \rightarrow \infty$ . The anisotropy parameter of expansion and anisotropy measure of fluid become

$$\Delta = 2 \frac{(m-1)^2}{(m+2)^2}, \quad (41)$$

$$\frac{(\delta - \gamma)}{\omega} = \frac{c(m-1)(m+2)^3}{(2m+1)(3m^2+3m-6-2ln)}. \quad (42)$$

We see that the anisotropy of the expansion and that of DE are constant so that both model and DE fluid remain anisotropic. The deviation free EoS parameter is found to be  $1 - (2ln)/(3(-2 + m + m^2))$  which is a constant.  $\omega$  may begin in the quintessence or phantom region depending on the values of constants.

## 4.2 Model with $m = 1$

For this value of  $m$ , Eq.(19) implies that

$$A(t) = B(t) = a(t) \quad (43)$$

and the spacetime (1) becomes

$$ds^2 = dt^2 - a^2(t)(dx^2 + dy^2 + dz^2) \quad (44)$$

which is the spatially flat FRW metric representing homogeneous and isotropic universe. Using Eq.(43) in (20), we have

$$3n \frac{\dot{a}^2}{a^2} - \frac{2\beta}{a^4} = 0 \quad (45)$$

which yields

$$a(t) = \sqrt{2}(\sqrt{\frac{2\beta}{3n}}t + c)^{1/2}. \quad (46)$$

The Hubble parameter and expansion scalar for this model become

$$H = \frac{\dot{a}}{a} = \frac{\beta}{(2\beta t + c\sqrt{6n\beta})}, \quad (47)$$

$$\Theta = 3H = \frac{3\beta}{(2\beta t + c\sqrt{6n\beta})} \quad (48)$$

while the energy density is

$$\rho = \frac{3\beta(2-n)}{8\beta t^2 + 12nc^2 + 8ct\sqrt{6n\beta}}. \quad (49)$$

We find that Hubble parameter and expansion scalar are constant at initial epoch i.e, at  $t = 0$  and decrease with the increase in time. Hence the universe is expanding for the earlier times of the universe. The energy density is also constant for the earlier times of the universe and decreases with the increase in time. The deviation free EoS parameter is  $1/3$  which represents the radiation dominated phase of the early universe [23, 34]. Equation (46) implies that for the radiation dominated universe  $a(t) \propto t^{1/2}$  [35, 36], the model represents expanding universe. The anisotropy parameter of the expansion is zero since Eq.(44) is an isotropic model. The deviation parameters  $\delta(t)$ ,  $\gamma(t)$  are constants and hence the anisotropy of the DE  $\frac{\delta-\gamma}{\omega} = \frac{6n}{(2-n)}$  is also constant. The anisotropy of the DE vanishes if we choose the dimensionless constant  $n$  to be zero.

## 5 Summary and Conclusion

In this paper we have constructed Bianchi  $I$  cosmological model with magnetized anisotropic DE fluid having anisotropic EoS. The deviation parameters  $\delta$  and  $\gamma$  are obtained by assuming that conservation equation of DE consists of two separate conserved parts. The exact solution of the field equations is obtained using the condition that expansion scalar  $\Theta$  is proportional to  $\sigma$ . We have discussed some physical aspects of the model both in the presence and absence of magnetic field.

The component of magnetic field reduces the energy density, expansion, shear scalar and Hubble parameters. The expansion in the universe is found to be infinite at the initial epoch which decreases with the increase in time. In the absence of magnetic field, a similar behavior of expansion is observed for  $n < \frac{3m^2+3m-6}{l}$ , however, if  $n > \frac{3m^2+3m-6}{l}$  then the universe starts expanding at  $T = 0$  and expands indefinitely as  $T \rightarrow \infty$ . The anisotropy measure of the DE is dynamical and found to be finite for both earlier and later times of the universe. The isotropic DE can be recovered by choosing  $n$  to be null, where  $n$  parameterizes the deviation parameters. The universe model does not approach to isotropy since anisotropy parameter of expansion  $\Delta$  is found to be constant with and without magnetic field. For  $m = 1$ , the Bianchi  $I$

model is reduced to spatially flat FRW metric which represents homogeneous and isotropic universe for earlier times. For this case, the deviation free EoS parameter is found to be  $1/3$  which represents the radiation dominated phase of the early universe [23, 34].

## References

- [1] Perlmutter, S. et al.: *Astrophys. J.* **483**(1997)565; Perlmutter, S. et al.: *Nature* **391**(1998)51; Perlmutter, S. et al.: *Astrophys. J.* **517**(1999)565.
- [2] Riess, A.G. et al.: *Astron. J.* **116**(1998)1009.
- [3] Bennett, C.L. et al.: *Astrophys. J. Suppl.* **148**(2003)1; Spergel, D.N. et al.: *Astrophys. J. Suppl.* **148**(2003)175.
- [4] Verde, L. et al.: *Mon. Not. R. Astron. Soc.* **335**(2002)432; Hawkins, E. et al.: *Mon. Not. Roy. Astr. Soc.* **346**(2003)78; Abazajian, et al.: *Phys. Rev.* **D69**(2004)103501.
- [5] Hinshaw, G. et al.: *Astrophys. J. Suppl.* **180**(2009)225.
- [6] Weinberg, S.: *Rev. Mod. Phys.* **61**(1989)1; Peebles, P. J. E. and Ratra, B.: *Rev. Mod. Phys.* **75**(2003)559.
- [7] Sahni, V. and Starobinsky, A.A.: *Int. J. Mod. Phys.* **D9**(2000)373; Sahni, V.: *Lect. Notes Phys.* **653**(2004)141; Padmanabhan, T.: *Gen. Relativ. Grav.* **40**(2008)529.
- [8] Caldwell, R.R.: *Phys. Lett.* **B23**(2002)545; Nojiri, S. and Odintsov, S.D.: *Phys. Lett.* **B147**(2003)562; *ibid.* **B1**(2003)565.
- [9] Sen, A.: *JHEP* **48**(2002)204; Padmanabhan, T.: *Phys. Rev.* **D66**(2002)021301; Padmanabhan, T. and Choudhury, T.R.: *Phys. Rev.* **D66**(2002)081301.
- [10] Feng, B., Wang, X.L. and Zhang, X.M.: *Phys. Lett.* **B607**(2005)35; Guo, Z.K. et al.: *Phys. Lett.* **B608**(2005)177.

- [11] Kamenshchik, A., Moschella, U. and Pasquier, V.: Phys. Lett. **B511**(2001)265; Bento, M.C., Bertolami, O. and Sen, A.A.: Phys. Rev. **D66**(2002)043507.
- [12] Wang, B., Gong, Y. G. and Abdalla, E.: Phys. Lett. **B624**(2005)141; Setare, M. R.: Phys. Lett. **B642**(2006)1; Hu, B. and Ling, Y.: Phys. Rev. **D73**(2006)123510; Kim, H., Lee, H. W. and Myung, Y. S.: Phys. Lett. **B632**(2006)605; Setare, M. R.: JCAP **0701**(2007)023.
- [13] Li, M.: Phys. Lett. **B603**(2004)1; Deffayet, C., Dvali, G.R. and Gabadadze, G.: Phys. Rev. **D65**(2002)044023.
- [14] Rodrigues, D.C.: Phys. Rev. **D77**(2008)023534.
- [15] Koivisto, T. and Mota, D.F.: JCAP **806**(2008)18.
- [16] Koivisto, T. and Mota, D.F.: Astrophysical J. **679**(2008)1.
- [17] Mota, D. F. et al.: Mon. Not. Roy. Astron. Soc. **382**(2007)793.
- [18] Akarsu, O. and Kilinc, C.B.: Gen. Relativ. Grav. **42**(2010)1.
- [19] Akarsu, O. and Kilinc, C.B.: Gen. Relativ. Grav. **42**(2010)763.
- [20] Madsen, M.S.: Mon. Not. R. Astron. Soc. **237**(1989)109.
- [21] King, E.J. and Coles, P.: Class. Quantum Grav. **24**(2007)2061.
- [22] Milaneschi, E. and Fabbri, R.: Astron. Astrophys. **151**(1985)7.
- [23] Jacobs, K.C.: Astrophys. J. **155**(1969)379.
- [24] Roy, S.R., Singh, J.P. and Narin, S.: Aust. J. Phys. **38**(1985)239.
- [25] Carroll, S.M. Hoffman, M. and Trodden, M.: Phys. Rev. **D68**(1992)023509.
- [26] Collins, C.B. and Hawking, S.W.: Astrophys. J. **180**(1973)317.
- [27] Thorne, K.S.: Astrophys. J. **148**(1967)51.
- [28] Kantowski, R. and Sachs, R.K.: J. Math. Phys. **7**(1966)433.
- [29] Kristian, J. and Sachs, R.K.: Astrophys. J. **143**(1966)379.

- [30] Collins, C.B.: Phys. Lett. **A60**(1977)397.
- [31] Roy, S.R. and Banerjee, S.K.: Class. Quantum Grav. **11**(1995)1943.
- [32] Bali, R. and Kumawat, P.: Phys. Lett. **B665**(2008)332.
- [33] Bali, R. Banerjee, R. and Banerjee, S.K.: Astrophys. Space Sci. **317**(2008)21.
- [34] Singh, C.P. and Kumar, S.: Astrophys. Space Sci. **310**(2007)31.
- [35] Grøn, Ø.: Am. J. Phys. **54**(1986)1.
- [36] Padmanabhan, T.: *Structure Formation in the Universe* (Cambridge University Press, 1993).